OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Core Mathematics 1

4721

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 May 2009
Afternoon

Duration: 1 hour 30 minutes

4721

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

No calculator can be used for this paper

1 Given that $y = x^5 + \frac{1}{x^2}$, find

(i) $\frac{dy}{dx}$, **[3]**

(ii) $\frac{d^2y}{dx^2}$. **[2]**

2 Express $\frac{8+\sqrt{7}}{2+\sqrt{7}}$ in the form $a + b\sqrt{7}$, where a and b are integers. **[4]**

3 Express each of the following in the form 3^n:

(i) $\frac{1}{9}$, **[1]**

(ii) $\sqrt[3]{3}$, **[1]**

(iii) $3^{10} \times 9^{15}$. **[2]**

4 Solve the simultaneous equations

$$4x^2 + y^2 = 10, \qquad 2x - y = 4.$$ **[6]**

5 **(i)** Expand and simplify $(2x + 1)(x - 3)(x + 4)$. **[3]**

(ii) Find the coefficient of x^4 in the expansion of

$$x(x^2 + 2x + 3)(x^2 + 7x - 2).$$ **[2]**

6 **(i)** Sketch the curve $y = -\sqrt{x}$. **[2]**

(ii) Describe fully a transformation that transforms the curve $y = -\sqrt{x}$ to the curve $y = 5 - \sqrt{x}$. **[2]**

(iii) The curve $y = -\sqrt{x}$ is stretched by a scale factor of 2 parallel to the x-axis. State the equation of the curve after it has been stretched. **[2]**

7 **(i)** Express $x^2 - 5x + \frac{1}{4}$ in the form $(x - a)^2 - b$. **[3]**

(ii) Find the centre and radius of the circle with equation $x^2 + y^2 - 5x + \frac{1}{4} = 0$. **[3]**

8 Solve the inequalities

(i) $-35 < 6x + 7 < 1$, **[3]**

(ii) $3x^2 > 48$. **[3]**

9 A is the point $(4, -3)$ and B is the point $(-1, 9)$.

(i) Calculate the length of AB. **[2]**

(ii) Find the coordinates of the mid-point of AB. **[2]**

(iii) Find the equation of the line through $(1, 3)$ which is parallel to AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[4]**

10 **(i)** Solve the equation $9x^2 + 18x - 7 = 0$. **[3]**

(ii) Find the coordinates of the stationary point on the curve $y = 9x^2 + 18x - 7$. **[4]**

(iii) Sketch the curve $y = 9x^2 + 18x - 7$, giving the coordinates of all intercepts with the axes. **[3]**

(iv) For what values of x does $9x^2 + 18x - 7$ increase as x increases? **[1]**

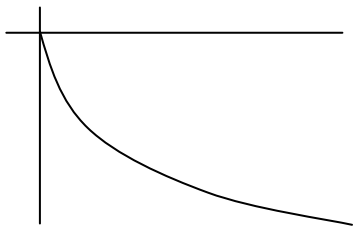
11 The point P on the curve $y = k\sqrt{x}$ has x-coordinate 4. The normal to the curve at P is parallel to the line $2x + 3y = 0$.

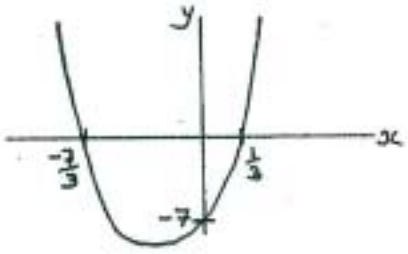
(i) Find the value of k. **[6]**

(ii) This normal meets the x-axis at the point Q. Calculate the area of the triangle OPQ, where O is the point $(0, 0)$. **[5]**

4721 Core Mathematics 1

1	(i)	$\frac{dy}{dx} = 5x^4 - 2x^{-3}$	**B1** **M1** **A1** **3**	$5x^4$ x^{-2} before differentiation or kx^{-3} in $\frac{dy}{dx}$ soi $-2x^{-3}$
	(ii)	$\frac{d^2y}{dx^2} = 20x^3 + 6x^{-4}$	**M1** **A1** **2** **5**	Attempt to differentiate their (i) – at least one term correct cao
2		$\frac{(8+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$ $= \frac{9-6\sqrt{7}}{4-7}$ $= -3+2\sqrt{7}$	**M1** **A1** **A1** **A1** **4** **4**	Multiply numerator and denominator by conjugate Numerator correct and simplified Denominator correct and simplified cao
3	(i)	3^{-2}	**B1** **1**	
	(ii)	$3^{\frac{1}{3}}$	**B1** **1**	
	(iii)	$3^{10} \times 3^{30}$ $= 3^{40}$	**M1** **A1** **2** **4**	3^{30} or 9^{20} soi
4		$y = 2x - 4$ $4x^2 + (2x-4)^2 = 10$ $8x^2 - 16x + 16 = 10$ $8x^2 - 16x + 6 = 0$ $4x^2 - 8x + 3 = 0$ $(2x-1)(2x-3) = 0$ $x = \frac{1}{2}$, $x = \frac{3}{2}$ $y =$ -3, $y =$ -1	**M1*** **A1** **M1dep*** **A1** **A1** **A1** **6** **6**	Attempt to get an equation in 1 variable only Obtain correct 3 term quadratic (aef) Correct method to solve quadratic of form $ax^2 + bx + c = 0$ $(b \neq 0)$ Correct factorisation oe Both x values correct Both y values correct or one correct pair of values www B1 second correct pair of values B1

5	**(i)**	$(2x^2-5x-3)(x+4)$ $=2x^3+8x^2-5x^2-20x-3x-12$ $=2x^3+3x^2-23x-12$	**M1** **A1** **A1** **3**	Attempt to multiply a quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term) Expansion with no more than one incorrect term
	(ii)	$2x^4+7x^4$ $=9x^4$ 9	**B1** **B1** **2** **5**	$2x^4$ or $7x^4$ soi www $9x^4$ or 9
6	**(i)**		**B1** **B1** **2**	One to one graph only in bottom right hand quadrant Correct graph, passing through origin
	(ii)	Translation Parallel to y-axis, 5 units	**B1** **B1** **2**	
	(iii)	$y=-\sqrt{\frac{x}{2}}$	**M1** **A1** **2** **6**	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen cao
7	**(i)**	$\left(x-\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+\frac{1}{4}$ $=\left(x-\frac{5}{2}\right)^2-6$	**B1** **M1** **A1** **3**	$a=\frac{5}{2}$ $\frac{1}{4}-a^2$ cao
	(ii)	$\left(x-\frac{5}{2}\right)^2-6+y^2=0$ Centre $\left(\frac{5}{2},0\right)$ Radius = $\sqrt{6}$	 **B1** **B1** **B1** **3** **6**	 Correct x coordinate Correct y coordinate

8	**(i)**	$-42 < 6x < -6$	**M1**	2 equations or inequalities both dealing with all 3 terms
		$-7 < x < -1$	**A1**	-7 and -1 seen oe
			A1 3	$-7 < x < -1$ (or $x > -7$ and $x < -1$)
	(ii)	$x^2 > 16$	**B1**	±4 oe seen
		$x > 4$	**B1**	$x > 4$
		or $x < -4$	**B1 3**	$x < -4$ not wrapped, not 'and'
			6	
9	**(i)**	$\sqrt{(^-1-4)^2+(9-^-3)^2}$	**M1**	Correct method to find line length using Pythagoras' theorem
		$=13$	**A1 2**	cao
	(ii)	$\left(\frac{4+^-1}{2}, \frac{^-3+9}{2}\right)$	**M1**	Correct method to find midpoint
		$\left(\frac{3}{2}, 3\right)$	**A1 2**	
	(iii)	Gradient of $AB = -\frac{12}{5}$	**B1**	
		$y-3=-\frac{12}{5}(x-1)$	**M1**	Correct equation for line, any gradient, through (1, 3)
		$12x+5y-27=0$	**A1**	Correct equation in any form with gradient simplified
			A1 4	$12x+5y-27=0$
			8	
10	**(i)**	$(3x+7)(3x-1)=0$	**M1**	Correct method to find roots
		$x=-\frac{7}{3}, x=\frac{1}{3}$	**A1**	Correct factorisation oe
			A1 3	Correct roots
	(ii)	$\frac{dy}{dx}=18x+18$	**M1**	Attempt to differentiate y
		$18x+18=0$	**M1**	Uses $\frac{dy}{dx}=0$
		$x=-1$	**A1**	
		$y=-16$	**A1 ft 4**	
	(iii)		**B1**	Positive quadratic curve
			B1	y intercept (0, -7)
			B1 3	Good graph, with correct roots indicated and minimum point in correct quadrant
	(iv)	$x > -1$	**B1 1**	
			11	

11 (i)	Gradient of normal = $-\frac{2}{3}$	**B1**	
	$\frac{dy}{dx} = \frac{1}{2}kx^{-\frac{1}{2}}$	**M1***	Attempt to differentiate equation of curve
		A1	$\frac{1}{2}kx^{-\frac{1}{2}}$
	When $x = 4$, $\frac{dy}{dx} = \frac{k}{4}$	**M1dep***	Attempt to substitute $x = 4$ into their $\frac{dy}{dx}$ soi
	$\therefore \frac{k}{4} = \frac{3}{2}$	**M1dep***	Equate their gradient expression to negative reciprocal of their gradient of normal
	$k = 6$	**A1 6**	cao
(ii)	P is point (4, 12)	**B1 ft**	
	Q is point (22, 0)	**M1**	Correct method to find coordinates of Q
		A1	Correct x coordinate
	Area of triangle $= \frac{1}{2} \times 12 \times 22$	**M1**	Must use y coordinate of P and x coordinate of Q
	$= 132$ sq. units	**A1 5**	
		11	